# Estimation of a Radiative Property of Scattering and Absorbing Media<sup>1</sup>

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This study proposes a simple estimation method for the albedo, which is one of the radiative properties of scattering and absorbing media. The method enables us to determine the albedo from the measured emittance of the medium without a complicated inverse analysis, using the relation that the emittance of the optically thick medium depends only on the albedo when the scattering occurring in the medium is assumed to be isotropic. This study also deals with a measurement method for the emittance of the scattering and absorbing medium, which is necessary to estimate the albedo, and demonstrates the validity and the usefulness of both the measurement method for the emittance and the estimation method for the albedo.

**KEY WORDS:** albedo; emittance; radiation transfer; radiative property; scattering.

# **1. INTRODUCTION**

Porous and fibrous media made of ceramics are commonly used for many industrial applications, including thermal insulation. In porous and fibrous media, radiation plays an important role in heat transfer. It is important to evaluate the radiation transfer in the media to improve their performance.

Propagation of radiation through the medium is scattered and/or absorbed by the solid medium. The radiation transfer can be evaluated by the radiation transfer equation. However, the data on radiative properties which characterize the radiation transfer cannot be easily obtained.

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Several estimation methods for the radiative properties have been developed. Most use some types of inverse method because the radiative properties cannot be directly measured [1]. However, since inverse methods usually require a complicated analysis, it is difficult to determine the radiative properties using such estimation methods.

The authors have presented an estimation method for radiative properties [2]. In the method, the radiative properties are determined from the measured energy of radiation emitted by planar scattering and absorbing media which are heated from the back side. However, this method also requires a complicated inverse analysis.

In this study, we present a simple estimation method for the albedo, which is one of the important radiative properties and significantly affects the radiation transfer in the scattering and absorbing medium. This is a modified method of our previous one and enables us to determine the albedo without the complicated inverse analysis.

The estimation method is based on the fact that emitted radiant energy does not depend on the medium thickness when the medium is optically thick. In this method, assuming that the scattering in the medium is isotropic, we determine the albedo from the measured emittance of a scattering and absorbing medium.

# 2. EMITTANCE AND ALBEDO

In this study, emittance of the scattering and absorbing medium is defined as the ratio of radiation energy emitted hemispherically from a planar isothermal medium (Fig. 1) to the radiant energy of the blackbody



Fig. 1. Schematic diagram of the physical system.

#### Estimation of a Radiative Property

at the same temperature as that of the medium. We investigate the effect of the albedo on emittance.

The radiation transfer equation for scattering and absorbing media governing the physical system (Fig. 1) is represented by the following equations assuming azimuthal symmetry:

$$\frac{\cos\theta}{\beta} \frac{dI(x,\theta)}{dx} + I(x,\theta)$$
$$= (1-\omega) I_{\rm b}(T_0) + \frac{\omega}{2} \int_0^{\pi} I(x,\theta') \Phi(\theta' \to \theta) \sin\theta' d\theta' \tag{1}$$

where  $I(x, \theta)$  is the radiation intensity,  $\theta$  the polar angle.  $I_{b}(T_{0})$  is the blackbody radiation intensity at the medium temperature,  $T_{0}$ , while  $\beta$ ,  $\omega$ , and  $\Phi(\theta' \rightarrow \theta)$  are the extinction coefficient, the albedo, and the phase functions, respectively, and  $\theta'$  and  $\theta$  the directions of the incident and scattering radiation, respectively.

The boundary conditions are

$$I(0, \theta) = I_{\rm b}(T, \theta) \qquad (0 < \cos \theta \le 1) \tag{2}$$

$$I(L, \theta) = I_{b}(T_{\ell}) \qquad (-1 \le \cos \theta < 0) \tag{3}$$

where  $T_{i}$  is the ambient temperature.

Assuming the isotropic scattering, i.e.,  $\Phi(\theta' \to \theta) = 1$ , and applying the two-flux methods for the radiation transfer, the formal solution for the forward component of the averaged radiation intensity  $J^+(\tau)$  and the backward component  $J^-(\tau)$  can be, respectively, expressed as follows.

For  $\cos \theta > 0$ 

$$J^{+}(\tau) = J^{+}(0) E_{2}(\tau) + \int_{0}^{\tau} \left[ (1-\omega) I_{b}(T_{0}) + \frac{\omega}{2} (J^{+}(t) + J^{-}(t)) \right] E_{1}(\tau-t) dt$$
(4)

For  $\cos \theta < 0$ 

$$J_{-}(\tau) = J_{-}(\tau_{0}) E_{2}(\tau_{0} - \tau) + \int_{\tau}^{\tau_{0}} \left[ (1 - \omega) I_{b}(T_{0}) + \frac{\omega}{2} (J^{+}(t) + J_{-}(t)) \right] E_{1}(t - \tau) dt \quad (5)$$

where  $\tau$  is the optical coordinate shown in Fig. 1 and  $E_n$  is the *n*th exponential integral function. The boundary conditions at x = 0 ( $\tau = 0$ ) and  $x = L(\tau = \tau_0)$  are

$$J^{+}(0) = I_{b}(T_{\gamma})$$
(6)

$$J_{-}(\tau_{0}) = I_{b}(T_{\infty}) \tag{7}$$

respectively. The detail derivation of these equations is shown in the literature [2].

According to the above-mentioned definition, the emittance can be calculated by the following equation under the condition of  $I_b(T_{\perp}) = 0$ .

$$\varepsilon = 2\pi \cdot \int_0^{\pi^2} I(L, \theta) \cos \theta \, d\theta / \pi I_{\rm b}(T_0) \tag{8}$$

Using the averaged radiation intensities, Eq. (8) can be rewritten as

$$\varepsilon = \frac{1}{I_{\rm b}(T_0)} \left\{ \int_0^{\tau_0} \left[ (1 - \omega) I_{\rm b}(T_0) + \frac{\omega}{2} (J^+(t) + J^-(t)) \right] E_2(\tau_0 - t) dt \right\}$$
(9)

We solved Eqs. (4) and (5) numerically using a finite differential procedure and calculated the emittance substituting the obtained averaged intensities into Eq. (9). Figure 2 shows the emittance against the optical thickness of the medium. The emittance does not depend on the medium temperature.



Fig. 2. Effect of the optical thickness of the medium on emittance.



Fig. 3. Relation between albedo and emittance for optically thick media.

In the region where the optical thickness is small, emittance increases with increasing optical thickness. This is because the radiant energy generated in the medium increases with increasing thickness. In the region where the optical thickness is sufficiently large, although the amount of the generated radiant energy is large, the radiation generated in the deep region is attenuated before arriving at the medium surface. Therefore, emittance becomes constant as the optical thickness of the medium increases.

The constant value of the emittance of the medium having large optical thickness depends on only albedo of the medium, when the isotropic scattering is assumed. Figure 3 shows the relation between the albedo and the emittance, which is derived from Fig. 2.

The optical thickness of a planar scattering and absorbing medium can be obtained by measuring the directional transmittance at the direction normal to the medium. Therefore, using the relation shown in Fig. 3, one can easily derive the albedo from the measured emittance for the medium having sufficiently large optical thickness, e.g., more than 20.

### 3. MEASUREMENT OF EMITTANCE

#### 3.1. Equipment

The emittance of the scattering and absorbing medium can be obtained by measuring the radiant energy emitted from the isothermal sample. We can refer to some measurement methods for the radiant energy from the isothermal medium in the previous studies. In one method, a sample of the scattering and absorbing medium is heated isothermally in a furnace, and then it is quickly covered by a cold tube so that the tube wall obstructs the radiation emitted from the furnace wall. The radiation emitted from the sample is measured through the tube before the tube is heated [3]. In another method, the isothermally heated sample in a furnace is quickly removed out of the furnace and the radiation emitted by the sample is measured before it is cooled [4].

These measurements must be performed in a short period. Therefore the methods are not suitable in monochromatic measurement, which usually requires some time. Here, we present a new and simple measurement method for radiant energy emitted by an isothermal sample of the scattering and absorbing medium.

In this method, the sample is heated by hot air exhausted by a furnace and the radiation emitted from the sample is monochromatically measured in a static condition. Figure 4 shows the measurement arrangement for radiant energy emitted from a planar sample. The two hot-air blowers having electric heaters are placed at the bottom of the furnace. After the hot air going into the furnace is well mixed by stainless-steel meshes, the hot air passes through the strainer and goes out of the furnace. Temperature distributions of the exhausted host air are shown in Fig. 5. It is found that the temperature distribution is almost constant in the region of  $40 \times 40$  (area)  $\times 80$  mm (height) above the outlet.



Fig. 4. Measurement arrangement for radiant energy emitted by a planar scattering and absorbing medium.



Fig. 5. Temperature distribution of the exhausted air.

To evaluate the utility of the healing equipment, a ceramic sample of the scattering and absorbing medium was placed in the isothermal region and heated isothermally. Temperature fluctuation of the air near the sample was measured by Cu-constantan thermocouples 0.1 mm in diameter during the radiant energy measurements. It was found that the fluctuation was within  $\pm 8$  K and it did not affect the radiation energy emitted from the sample. This is because the sample temperature did not fluctuate as much as the surrounding air due to its large heat capacity.

To obtain the emittance defined by Eq. (8) we have to measure all radiation energy emitted hemispherically by the sample. This requires complicated measurement equipment. However, if we need the emittance of porous or fibrous media, the measurement equipment would be simplified. This is because the directional emittance at the normal direction can be substituted for the present defined emittance due to their isotropic emission of radiation. In this study, we measure only the normally emitted radiation. A copper plate was used for the reference blackbody. The plate surface is coated with candle soot and heated isothermally on the back side using an electric heater.

#### 3.2. Data Reduction

Radiant energy is measured with a monochromator having a thermopile sensor of ambient temperature. Signal outputs corresponding to the radiant energy emitted from the sample and the reference blackbody,  $V_{\rm p}$  and  $V_{\rm b}$ , can be expressed, respectively, as

$$V_{\rm p} = c(\varepsilon E_{\rm b}(T_0) + \rho E_{\rm b}(T_{\rm y} - E_{\rm b}(T_{\rm y}))$$
<sup>(10)</sup>

$$V_{\rm b} = c(E_{\rm b}(T_0) - E_{\rm b}(T_{\rm c}))$$
(11)

where c is a proportionality constant,  $E_{\rm b}$  is the radiant energy emitted by the blackbody, and  $T_0$  and  $T_{\chi}$  are the sample and the ambient temperatures, respectively. The second term of the right-hand side of Eq. (10) shows the reflected background radiant energy and  $\rho$  is the reflectance of the sample. The last terms in each equation show the radiant energy emitted by the sensor.

Assuming that the emittance  $\varepsilon$  and the reflectance  $\rho$  of the mediums satisfy  $\varepsilon = 1 - \rho$  and using Eqs. (10) and (11), the emittance is determined by the following equation:

$$\varepsilon = V_{\rm p}/V_{\rm b} \tag{12}$$

# 4. DEMONSTRATION OF THE ESTIMATION METHOD FOR ALBEDO

To examine the validity and the usefulness of the present method, we measured the emittance of ceramic porous media and estimated their albedo. All the samples were made of a material which includes mainly  $Al_2O_3$ , but they had different pore sizes: 8, 18, and 26 pores per cm (PPCM). The samples' thicknesses were larger than 10 mm and their optical thicknesses were confirmed by transmittance measurements to be larger than 20, which is sufficiently large value.

The results of the emittance are shown in Fig. 6. Concerning these media, the pore size seems not to affect the emittance of the medium. In



Fig. 6. Emittance versus wavelength for ceramic porous media.



Fig. 7. Albedo versus wavelength for ceramic porous media.

Fig. 6, the results for a porous medium made of pure  $Al_2O_3$ , which were measured by Vader et al. [3], are also shown. Our results and those in Ref. 3 are in good agreement. It is considered that the measurement methods for the emittance of the scattering and absorbing medium are valid and useful. The albedo determined from the measured emittance using the relation in Fig. 3 is shown in Fig. 7.

When the emittance is high, the estimated values of the albedo are scattered, since the albedo is quite sensitive to the emittance. These results suggest that the estimation method is suitable for highly scattering media rather than absorbing media.

#### 5. SUMMARY

This study presented a simple estimation method for the albedo of scattering and absorbing media and demonstrated its validity and usefulness. Although the albedo was estimated assuming the isotropic scattering in this study, if the phase function could be obtained by any other method, the more accurate albedo would be derived by incorporating the phase function into the calculation for the relation between the albedo and the emittance.

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#### NOMENCLATURE

- *h* Height measured from the outlet of the furnace; m
- I Monochromatic radiation intensity;  $W \cdot m^{-3} \cdot sr^{-1}$

- $I_{\rm b}$  Monochromatic blackbody radiation intensity; W · m  $^{-3}$  · sr  $^{-1}$
- $J^+$  Forward component of averaged monochromatic radiation intensity; W · m<sup>-3</sup> · sr<sup>-1</sup>
- J Backward component of averaged monochromatic radiation intensity:  $W \cdot m^{-3} \cdot sr^{-1}$
- L Medium thickness; m
- $T_0$  Medium temperature; °C
- $T_{\gamma}$  Ambient temperature; °C
- *x* Geometrical coordinate; m
- $\beta$  Extinction coefficient; m<sup>-1</sup>
- $\varepsilon$  Emittance
- $\Phi$  Phase function
- $\theta$  Polar angle; rad
- $\rho$  Reflectance
- $\tau$  Optical coordinate (= $x\beta$ )
- $\tau_0$  Medium optical thickness (= $L\beta$ )
- $\omega$  Albedo

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